

Pendulum Motion

Aim: To determine the experimental value for the gravitational field vector, g .

Theory: This experiment is based on the theory that when a small pendulum swings with a small angle, the mass on the end of the swing performs a good approximation of simple harmonic motion. The period of a pendulum swing depends upon the length of the string and the rate of acceleration due to gravity.

The formula for the period of a swing:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

can be rearranged to give an expression for g as:

$$g = \frac{4\pi^2 l}{T^2}$$

where T = Period of the pendulum swing in seconds.

l = Length of pendulum in metres.

According to the Jacaranda Physics 2 HSC Course textbook and the Oxford University Press Physics in Context textbook, the average gravitational acceleration at sea-level is 9.80 ms^{-2} .

Hypothesis: The method to be used is expected to produce a value for the gravitational field vector, g , reasonably close to the published value. The longer the pendulum length, the closer the experimentally calculated value is expected to come to the published value.

Equipment:

- Video Camera
- One metre ruler
- 1.20m string
- Plumb Bob
- Text book
- Boss head
- Clamp
- Retort Stand
- Oversize protractor
- Box
- Bench

Method:

1. Set up retort stand, boss head, clamp and text book so that the round end of the clamp sticks out over the end of the bench. **SAFETY: Ensure that the apparatus will not tip over.**
2. Set up protractor so that the pendulum swings directly in front of it.
3. Set up video camera so that it can view plumb bob swinging directly in front of protractor. Use box to adjust the high of camera. **SAFETY: Ensure that the power cable from the camera will not be tripped over and is not frayed.**
4. Attach string of approx 1.20m to round end of clamp and attach other end to plumb bob.
5. Measure actual length of string (in metres) from the knot at the clamp to the centre of the plumb bob.
6. Press record on the video camera.
7. Pull string back approximately 20-30° along a path parallel to the bench and release.
8. Allow plumb bob to complete ten back-and-forth oscillations then repeat step 6 to 8 twice more. **SAFETY: Do not walk in the path of the swinging pendulum.**
9. Once three tests have been completed for this string length, cut approximately 0.10-0.15m from the string and repeat steps 5 to 8 for five different string lengths ranging from 0.60m to 1.20m.
10. Download video footage of pendulum swinging.
11. Use frame by frame analysis to determine the period (in seconds) of the pendulum and maximum points on the pendulum swing.
12. Use the formula $g = \frac{4\pi^2 l}{T^2}$ to determine the value for the gravitational field vector, g, for each test.
13. Plot a graph of $4\pi^2 l$ over T^2 for each of the tests and apply a line of best fit to determine the experimentally determined value.

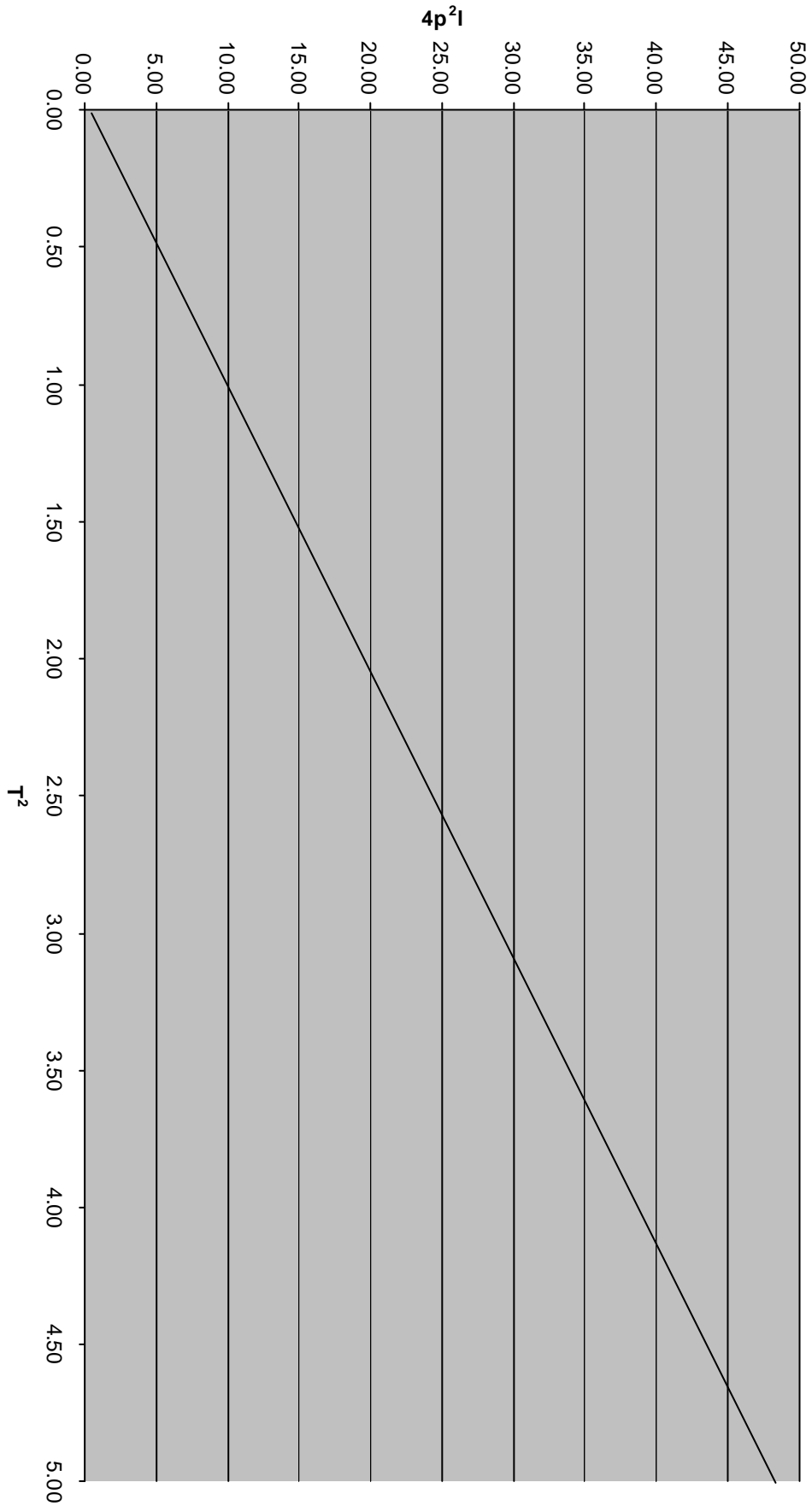
Results:**Table 1: Pendulum Lengths & Periods**

Test #	Pendulum Length (m)	Time for 10 Oscillations (s)	Av Swing Time (s)	g (ms ⁻²)
1	1.16	21.80	2.180	9.64
2	1.16	21.80	2.180	9.64
3	1.16	21.84	2.184	9.60
4	1.00	20.24	2.024	9.64
5	1.00	20.24	2.024	9.64
6	1.00	20.28	2.028	9.60
7	0.87	18.96	1.896	9.55
8	0.87	18.92	1.892	9.59
9	0.87	18.92	1.892	9.59
10	0.78	17.92	1.792	9.59
11	0.78	17.88	1.788	9.63
12	0.78	17.88	1.788	9.63
13	0.62	16.12	1.612	9.42
14	0.62	16.08	1.608	9.47
15	0.62	16.08	1.608	9.47
			AVERAGE	9.58

Table 2 – Average Values of g for each Pendulum Length

String Length (m)	Average g (ms ⁻²)
1.16	9.62
1.00	9.62
0.87	9.58
0.78	9.62
0.62	9.45

Graph 1 - $4p^2l$ against T^2

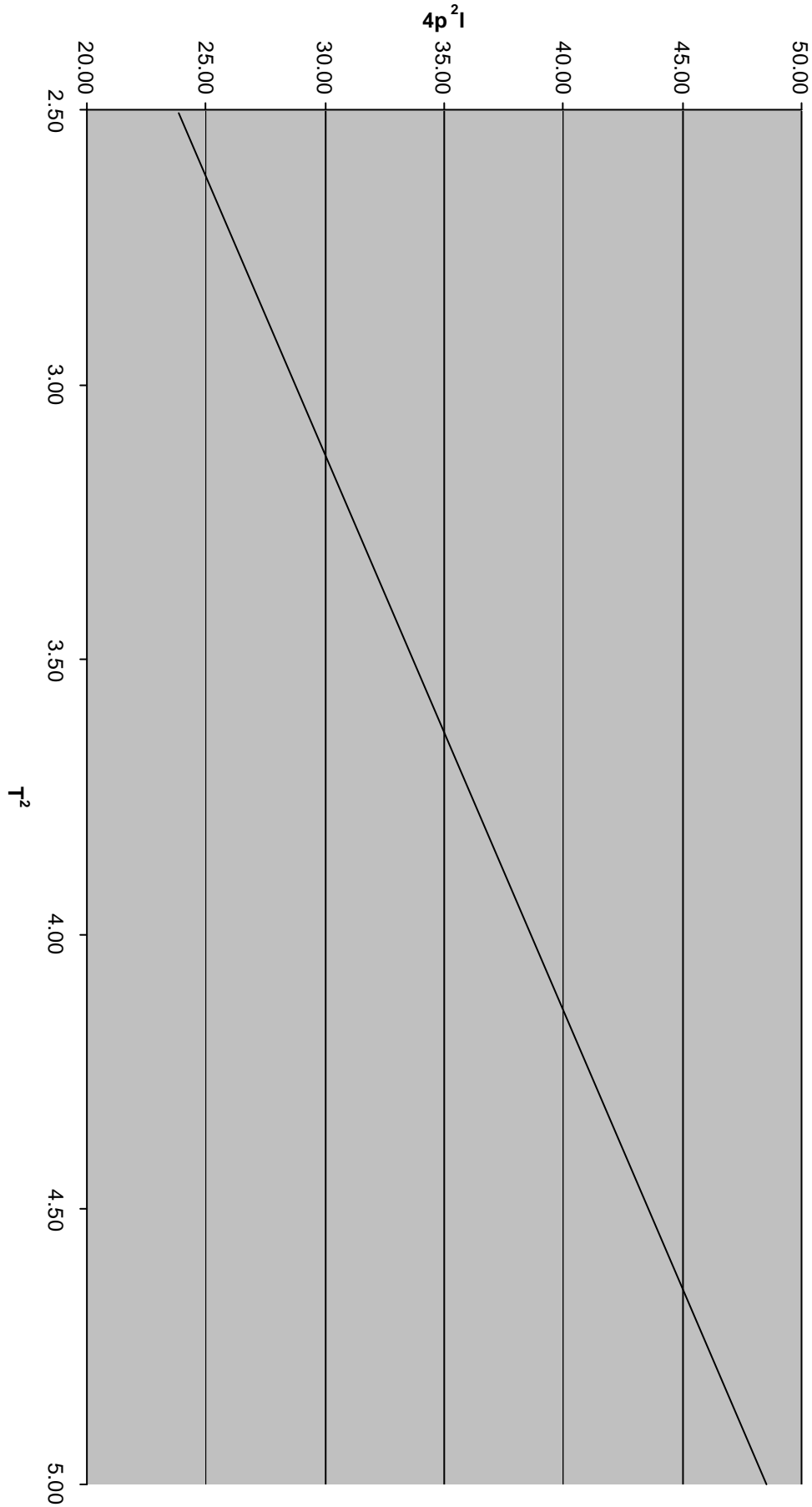


Pendulum Motion

Table 3: Components of Formula

Test #	Pendulum Length (m)	$4\pi^2 l$	T^2
1	1.16	45.79	4.75
2	1.16	45.79	4.75
3	1.16	45.79	4.77
4	1.00	39.48	4.10
5	1.00	39.48	4.10
6	1.00	39.48	4.11
7	0.87	34.35	3.59
8	0.87	34.35	3.58
9	0.87	34.35	3.58
10	0.78	30.79	3.21
11	0.78	30.79	3.20
12	0.78	30.79	3.20
13	0.62	24.48	2.60
14	0.62	24.48	2.59
15	0.62	24.48	2.59

Graph 2 - $4p^2l$ against T^2 (Zoomed Up)

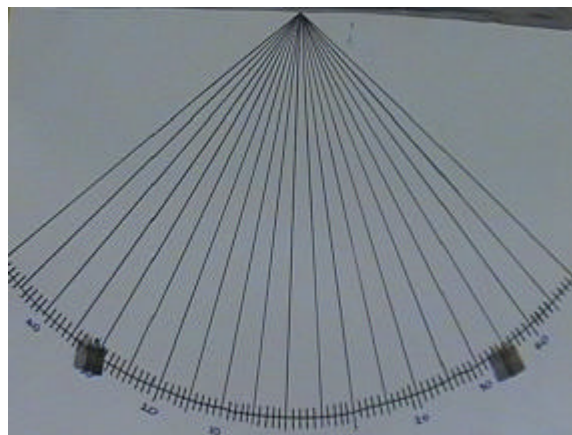


Pendulum Motion

The gradient of the line of best fit is 9.58.
The average of all the values was 9.58.

Calculations: See Appendix 1a.

Analysis: Over the course of the experiment there were many possible sources of error which could have contributed to the overall error percentage. The experimentally determined value of 9.58 was 2.2% off the accepted published value (see Appendix 1b) and this was due to a number of possible sources of error.



Picture 1 – Overlay of Max Points

The first was the assumption that the pendulum was completing a full swing through exactly the same angle for every oscillation. When the motion of the pendulum was analysed using a frame-by-frame analysis, it became apparent that the swing size was decreasing by approximately 0.7% over each oscillation. Picture 1 demonstrates this by showing the maximum points for the first and tenth swings. From this figure we can assume that air resistance was 'slowing' down the pendulum by 0.7% on each oscillation. This then means that the period of each pendulum was increased by 0.7% on each swing. Table 4 shows the re-calculated values for g allowing for the 0.7% affect of air resistance.

Table 4 – Re-calculated Times (see Appendix 1c)

Test #	String Length (m)	Av Swing Time (s)	g (ms^{-2})	Adjusted Time (s)	Adjusted g (ms^{-2})
1	1.16	2.180	9.64	2.165	9.77
2	1.16	2.180	9.64	2.165	9.77
3	1.16	2.184	9.60	2.169	9.74
4	1.00	2.024	9.64	2.010	9.77
5	1.00	2.024	9.64	2.010	9.77
6	1.00	2.028	9.60	2.014	9.74
7	0.87	1.896	9.55	1.883	9.69
8	0.87	1.892	9.59	1.879	9.73
9	0.87	1.892	9.59	1.879	9.73
10	0.78	1.792	9.59	1.779	9.73
11	0.78	1.788	9.63	1.775	9.77
12	0.78	1.788	9.63	1.775	9.77
13	0.62	1.612	9.42	1.601	9.55
14	0.62	1.608	9.47	1.597	9.60
15	0.62	1.608	9.47	1.597	9.60
		AVERAGE	9.58	AVERAGE	9.72

Using these recalculated values for g , the average value for g is now only 0.8% off the accepted published value (see Appendix 1d).

There were also other possible sources for error. The first of which is the one metre ruler used for measuring the length of pendulum swing. The ruler only measured to the nearest 5mm, therefore making the absolute error on all measurements made using the metre ruler $\pm 2.5 \times 10^{-3}\text{m}$. This equates to a relative error percentage of 0.2% on the longest length of string up to an error percentage of 0.4% on the shortest length of string.

Another possible margin for error is the timing made by the video camera. The video camera records 25 frames per second, which equates to 0.04 seconds per frame. The smallest increment of time measurement on the video camera is one frame so the absolute error on all time recordings is ± 0.02 seconds. This is the equivalent to a 0.09% relative error on the longest time for ten oscillations, up to an error margin of 0.13% on the shortest time for ten oscillations. (See Appendix 1e)

It was also noted that the longer the length of the pendulum, the closer the experimental value of g came to the published value. Table 5 compares the calculated values of g for each pendulum length with the possible margin for error.

Table 5 – Margins for Error

String Length (m)	Experimental Results		Adjusted Results		Measurement Error %
	Average g (ms^{-2})	% Error	Av Adjusted g (ms^{-2})	% Error	
1.16	9.62	1.8	9.76	0.4	0.31
1.00	9.62	1.8	9.76	0.4	0.35
0.87	9.58	2.2	9.72	0.8	0.40
0.78	9.62	1.9	9.75	0.5	0.44
0.62	9.45	3.6	9.59	2.2	0.53

As Table 5 shows, for the majority of the tests the difference between the adjusted value for g and the published value can be mostly accounted for by the limit of reading on the equipment (See Appendix 1f). All, that is, except the pendulum lengths of 0.87m and 0.62m. This source of error could have occurred if the pendulum did not swing parallel to the bench but swung in a more elliptical shape. Whilst all care was taken to ensure that the pendulum swung parallel to the bench there is always the possibility that it did swing slightly off course. Unfortunately, due to the nature of a video camera, which records in only two dimensions, small variations in path were not detectable on the frame-by-frame analysis.

The 0.62m pendulum was out by the furthest of all the tests and the error due to measurement limitations doesn't add up to the overall error percentage. It could therefore be assumed that the test using the 0.62m pendulum length is the one most likely to have travelled in an elliptical path.

The method used during this experiment gave reliable and valid results. The results were reliable as they were repeated and similar results were obtained. That is, the results obtained with each string length were similar except for the 0.62m string length due to the reasons given above.

The experiment was also valid as it fairly tested the hypothesis and all variable that needed to be controlled were so. One such example of a controlled variable was the altitude that the pendulum swung, which was kept constant throughout the experiment. There were no unaccountable human errors, such as reaction time when using a stopwatch and any errors were consistent ones that could be mathematically accounted for.

The reliability of the information taken from secondary sources, for example the published value for g , was also very reliable. The textbooks from which the information was obtained are printed by very reputable companies. That, combined with the fact that both sources had the same data allows them to be considered reliable.

Conclusion: The experimentally determined value for the gravitational field vector, g , was 9.58 ms^{-2} . The adjusted value for g was 9.72 ms^{-2} . As hypothesised, the longer the string, the closer the values came to the published values.